10.

$a_2^2 = a_1 a_3$

$\left(\frac{1}{x-2}\right)^2 = \frac{1}{x+2} \cdot \frac{1}{x-4}$

$\frac{1}{(x-2)^2} = \frac{1}{(x+2)(x-4)} \quad / \cdot (x-2)^2 (x+2)(x-4)$

$(x+2)(x-4) = (x-2)^2$

$x^2 - 4x + 2x - 8 = x^2 - 4x + 4$

$2x = 12$

$\boxed{x = 6}$

Niz $\frac{1}{8}; \frac{1}{4}, \frac{1}{2}, \ldots$ $\boxed{q = 2}$ $\boxed{a_1 = \frac{1}{8}}$

11.

$\log_4 9 = \frac{\log_2 9}{\log_2 4} = \frac{\log_2 3^2}{\log_2 2^2} = \frac{2\log_2 3}{2(\log_2 2)} = \log_2 3$ ($\log_2 2 = 1$)

$\log_5 2 \cdot \log_9 25 = \frac{\log_2 2}{\log_2 5} \cdot \frac{\log_2 25}{\log_2 9} =$

$= \frac{\log_2 2}{\log_2 5} \cdot \frac{\log_2 5^2}{\log_2 3^2} = \frac{1 \cdot 2\log_2 5}{\log_2 5 \cdot 2\log_2 3} = \frac{1}{\log_2 3}$

$a_1 = \log_2 3$

$a_2 = \log_2 a$

$a_3 = \frac{1}{\log_2 3}$

$a_2^2 = a_1 a_3$

$(\log_2 a)^2 = \log_2 3 \cdot \frac{1}{\log_2 3}$

$(\log_2 a)^2 = 1 \quad / \sqrt{\ }$

$\log_2 a = \pm 1 \Rightarrow \boxed{a_1 = 2^{-1}}$ $\boxed{a_2 = 2}$

12.

1) $a_1 - a_2 = 35$, $a_3 - a_4 = 560$

$a_1 - a_1 q = 35$

$a_1 q^2 - a_1 q^3 = 560$

$a_1(1-q) = 35$

$a_1 q^2 (1-q) = 560$

$q^2 \cdot 35 = 560$

$q^2 = \frac{560}{35} = 16$

$q = \pm 4$

$\boxed{q_1 = -4}$ $\boxed{q_2 = 4}$

$q_1 = -4$

$a_1(1+4) = 35$

$a_1 = \frac{35}{5}$

$\boxed{a_1 = 7}$ $\boxed{q_1 = -4}$

$q_1 = 4$

$a_1(1-4) = 35$

$\boxed{a_1 = \frac{35}{-3}}$ $\boxed{q_1 = 4}$

2) $a_4 - a_2 = 18$

$a_5 - a_3 = 36$

$a_1 q^3 - a_1 q = 18$

$a_1 q^4 - a_1 q^2 = 36$

$a_1 q (q^2 - 1) = 18$

$a_1 q^2 (q^2 - 1) = 36$

$q \cdot 18 = 36$

$\boxed{q = 2}$

$a_1 \cdot 2 (4 - 1) = 18$

$a_1 \cdot 6 = 18$

$\boxed{a_1 = 3}$

3) $a_1 - a_3 + a_5 = -65$

$a_1 + a_7 = -325$

$a_1 - a_1 q^2 + a_1 q^4 = -65$

$a_1 + a_1 q^6 = -325$

$a_1 (1 - q^2 + q^4) = -65$

$a_1 (1 + q^6) = -325$

$\frac{a_1 (1+q^2)(1-q^2+q^4)}{a_1 (1-q^2+q^4)} = \frac{-325^{5}}{-65_{1}}$

$1 + q^2 = 5$

$q^2 = 4$

$q = \pm 2$

$\boxed{q_1 = -2}$ $a_1 (1 + 64) = -325$

$a_1 = \frac{-325}{65}$

$\boxed{(a_1)_1 = -5}$

$\boxed{q_2 = 2}$ $a_1 (1 + 64) = -325$

$a_1 = \frac{-325}{65}$

$\boxed{(a_1)_2 = -5}$

4) $a_1 + a_4 = \frac{7}{16}$

$a_3 - a_2 + a_1 = \frac{7}{8}$

$a_1 + a_1 q^3 = \frac{7}{16}$

$a_1 q^2 - a_1 q + a_1 = \frac{7}{8}$

$a_1 (1 + q^3) = \frac{7}{16}$

$a_1 (q^2 - q + 1) = \frac{7}{8}$

$\frac{a_1 (1+q)(q^2 - q + 1)}{a_1 (q^2 - q + 1)} = \frac{\frac{7}{16}^{2}}{\frac{7}{8}_{1}}$

$1 + q = \frac{1}{2}$

$q = \frac{1}{2} - 1$

$\boxed{q = -\frac{1}{2}}$

$a_1 \left(1 - \frac{1}{8}\right) = \frac{7}{16}$

$a_1 \cdot \frac{7}{8} = \frac{7}{16}$

$a_1 = \frac{\frac{7}{16}^{2}}{\frac{7}{8}_{1}}$ $\boxed{a = \frac{1}{2}}$

5) $a_1+a_2+a_3=31$

$a_1+a_3=26$ $\Rightarrow$ $a_2+26=31$

$\boxed{a_2=5}$

$a_1+a_1q+a_2q^2=31$

$a_1+a_1q^2=26$

$a_1(1+q+q^2)=31$

$a_1(1+q^2)=26$

$\frac{5}{q}(1+q+q^2)=31 \;/\cdot q$

$\frac{5}{q}(1+q^2)=26 \;/\cdot q$

$5+5q+5q^2=31q$

$5+5q^2=26q$

$5q^2-26q+5=0$

$5q^2-26q+5=0$

$q_{1/2}=\frac{26\pm\sqrt{676-100}}{10}$

$q_{1/2}=\frac{26\pm 24}{2}$

$q_1=\frac{50}{2}=25 \Rightarrow (a_1)_1=\frac{5}{25}=\frac{1}{5}$

$q_2=\frac{2}{2}=1 \Rightarrow (a_1)_2=\frac{5}{1}=5$

$a_1q=5$

$\boxed{a_1=\frac{5}{q}}$

6) $a_1+a_2+a_3=70$

$a_1a_2a_3=8000$

$a_1+a_1q+a_1q^2=70$

$a_1\cdot a_1q\cdot a_1q^2=8000$

$a_1(1+q+q^2)=70$ (1)

$a_1^3q^3=8000 \;/^{3}\sqrt{\;}$

$a_1q=20$

$\boxed{a_1=\frac{20}{q}}$

(1) $\frac{20}{q}(1+q+q^2)=70 \;/\cdot q$

$20+20q+20q^2=70q$

$20q^2-50q+20=0 \;/:10$

$2q^2-5q+2=0$

$q_{1/2}=\frac{5\pm\sqrt{25-16}}{4}=$

$=\frac{5\pm 3}{4}$

$\boxed{q_1=2 \Rightarrow (a_1)_1=10}$

$\boxed{q_2=\frac{1}{2} \Rightarrow (a_1)_2=40}$

$$\frac{11\pi}{12} = \frac{15-4}{12}\pi = \frac{\overset{5}{\cancel{15}}}{\cancel{12}_4}\pi - \frac{\overset{1}{\cancel{4}}\pi}{\cancel{12}_3} = \frac{5\pi}{4} - \frac{\pi}{3}$$

$$\cos\frac{11\pi}{12} = \cos\left(\frac{5\pi}{4} - \frac{\pi}{3}\right) = \cos\frac{5\pi}{4}\cos\frac{\pi}{3} + \sin\frac{5\pi}{4}\sin\frac{\pi}{3} =$$

$$= -\frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{-\sqrt{2}(1+\sqrt{3})}{4}$$

$$\sin\frac{11\pi}{12} = \sin\left(\frac{5\pi}{4} - \frac{\pi}{3}\right) = \sin\frac{5\pi}{4}\cos\frac{\pi}{3} - \cos\frac{5\pi}{4}\sin\frac{\pi}{3} =$$

$$= -\frac{\sqrt{2}}{2}\frac{1}{2} + \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} = \frac{-\sqrt{2}(1-\sqrt{3})}{4}$$

$$(**) = 2\left(-\frac{\sqrt{2}(1+\sqrt{3}}{4} + i\,\frac{-\sqrt{2}(1-\sqrt{3})}{4}\right) =$$

$$= \frac{\overset{1}{\cancel{2}}\sqrt{2}}{\cancel{4}_2}\left(-1-\sqrt{3} + i(\sqrt{3}-1)\right) =$$

$$a_{20} = \frac{\sqrt{2}}{2}\left(-1-\sqrt{3} + i(\sqrt{3}-1)\right)$$

$$|a_{20}| = \sqrt{\frac{\overset{1}{\cancel{2}}}{\cancel{4}_2}\left[(-1-\sqrt{3})^2 + (\sqrt{3}-1)^2\right]} =$$

$$= \sqrt{\frac{1}{2}\left[1+2\sqrt{3}+3+3-2\sqrt{3}+1\right]} =$$

$$= \sqrt{\frac{1}{2}\cdot 8\,4} = \sqrt{4} = 2$$

$$\boxed{|a_{20}| = 2}$$

8.

$a_1 = x+5$
$a_2 = 25-x$
$a_3 = 30+2x$

Vrijedi: $\frac{a_2}{a_1} = \frac{a_3}{a_2} = 2$ za geometrijski niz!

$$a_2^2 = a_1 a_3$$

$$(25-x)^2 = (x+5)(30+2x)$$

$$625 - 50x + x^2 = 30x + 2x^2 + 150 + 10x$$

ovo su zadaci uz V.zadatak geometrijski niz - test

9.

a_{n-1} a_n a_{n+1}

$1, x^2, x^2+72$

Za geometrijski niz uvijek $a_n^2 = a_{n-1} \cdot a_{n+1}$

$$(x^2)^2 = 1 \cdot (x^2+72)$$

$$x^4 - x^2 - 72 = 0$$

$$(x^2)_{1,2} = \frac{1 \pm \sqrt{1+288}}{2} = \frac{1 \pm 17}{2} \Rightarrow x_1^2 = 9$$

~~$x_2^2 = -8$~~

$$x_{1,2} = \pm\sqrt{9} = \pm 3$$

$$\left.\begin{array}{l} a_{n-1} = 1 \\ a_n = 9 \\ a_{n+1} = 81 \end{array}\right\} \Rightarrow q = 9$$

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$\boxed{a_1 = \frac{35}{-3}}$ $\boxed{q_1 = 4}$